Solutions to JEE MAIN – 6 | JEE 2024

PHYSICS

SINGLE CHOICE

1.(B)

A	B	C
0	0	0
0	1	0
1	0	0
1	1	1

2.(D) $\dfrac{\dfrac{\mu_0 i_1}{2\pi r} - \dfrac{\mu_0 i_2}{2\pi r}}{\dfrac{\mu_0 i_1}{2\pi r} + \dfrac{\mu_0 i_2}{2\pi r}} = \dfrac{20}{50} \quad \Rightarrow \quad \dfrac{i_1 - i_2}{i_1 + i_2} = \dfrac{2}{5} \quad \Rightarrow \quad \dfrac{i_1}{i_2} = \dfrac{7}{3}$

3.(C) Let V_L be x ; $V_C = 2x$, $V_R = 3x$; $V_s = \sqrt{V_R^2 + (V_C - V_L)^2}$

$100 = \sqrt{9x^2 + x^2} \Rightarrow x = 10\sqrt{10} \qquad \therefore \qquad V_R = 30\sqrt{10} = 94.86\ V$

4.(A) $\overline{C} = \overline{E} \times \overline{B}$ & $CB = E$ in magnitude.

5.(A) Wavelength of the emitted light is given by, $\dfrac{1}{\lambda} = R\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$. Put $n_1 = 2$ and $n_2 = 3$, and solve.

6.(C) The net magnetic field at the centre 'O' of the circular arc is $B_0 = \dfrac{\mu_0 I}{4\pi R} + \dfrac{3\mu_0 I}{8R} - \dfrac{\mu_0 I}{8\pi R}$

$= \dfrac{\mu_0 I}{8\pi R} + \dfrac{3\mu_0 I}{8R}, \qquad B_0 = \dfrac{\mu_0 I}{8R}\left(\dfrac{1+3\pi}{\pi}\right)$

7.(B) $\lambda = \dfrac{h}{\sqrt{2mE}} \Rightarrow \lambda \propto \dfrac{1}{\sqrt{E}} \qquad \Rightarrow \dfrac{\lambda_1}{\lambda_2} = \sqrt{\dfrac{E_2}{E_1}}$

$\dfrac{\lambda}{\left(\dfrac{\lambda}{2}\right)} = \sqrt{\dfrac{E_2}{E_1}} \Rightarrow E_2 = 4E_1$

Also, $E_1 = E$ & $E = E + \Delta E \quad \Rightarrow \quad E + \Delta E = 4E \qquad \Rightarrow \quad \Delta E = 3E$

8.(B) Rms current through resistor 'R_1' is $I_{1(rms)} = \dfrac{50}{\sqrt{2} \times 10\sqrt{2}} = \dfrac{5}{2} A$

$\therefore$ Average power developed in the resistor 'R_1' is

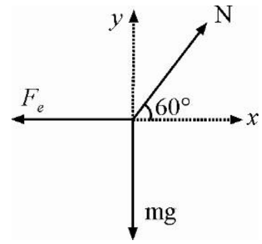
$P_1 = I_{1(rms)}^2 R_1 = \left(\dfrac{5}{2}\right)^2 10 = \dfrac{250}{4} = 62.50\ W$

9.(C) At path difference $\dfrac{\lambda}{6}$, phase difference is $\dfrac{\pi}{3}$.

$I = I_0 + I_0 + 2I_0 \cos\dfrac{\pi}{3} = 3I_0 \qquad I_{\max} = 4I_0$

So the required ratio is $\dfrac{3I_0}{4I_0} = 0.75$

10.(A) The bowl exerts a normal force N on each ball, directed along the radial line or at 60° above the horizontal. Consider the free-body diagram of the ball on the left with the electric force F_e applied.



$$\sum F_y = N\sin 60° - mg = 0, \ \Rightarrow N = mg/\sin 60°$$

$$\sum F_x = -F_e + N\cos 60° = 0, \ \Rightarrow \frac{Kq^2}{R^2} = N\cos 60° = \frac{mg}{\tan 60°} = \frac{mg}{\sqrt{3}}$$

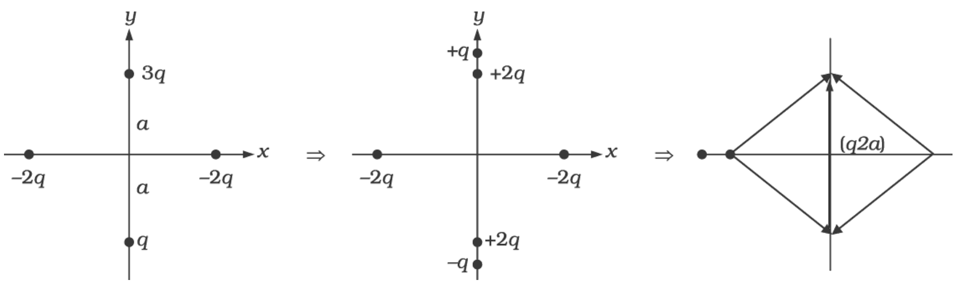
Thus, $q = R\left(\frac{mg}{K\sqrt{3}}\right)^{1/2}$

11.(C)

12.(A) $Q_1 = 4v$

$Q_2 = 4\times\frac{2v}{3} = \frac{8v}{3}$; $Q_3 = 4\times\frac{4v}{14} = \frac{8v}{7}$; $Q_1 > Q_2 > Q_3$

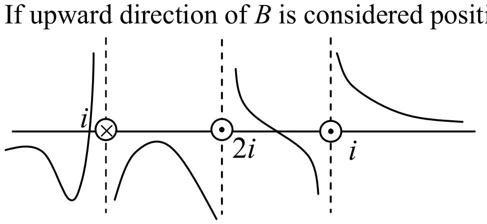
13.(A)



dipole moments by $-2q$ and $+2q$ cancel each other so

$\vec{P}_{net} = 2(aq)$ along y-axis ; $\vec{P}_{net} = 2aq\hat{j}$

14.(D) If upward direction of B is considered positive the variation can be shown as

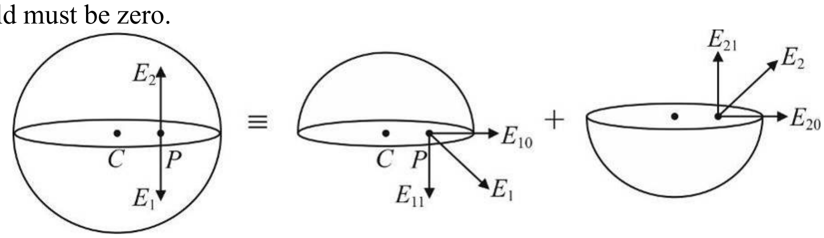


15.(C) $\frac{V}{R}e^{-t/RC} = \frac{V}{R}\left(1 - e^{-Rt/L}\right) \Rightarrow e^{-t/\sqrt{LC}} = 1 - e^{-t/\sqrt{LC}}$

$\Rightarrow e^{-t/\sqrt{LC}} = \frac{1}{2} \Rightarrow \frac{t}{\sqrt{LC}} = \ell n2 \Rightarrow t = \sqrt{LC}\ \ell n2 = RC\ \ell n2$

16.(C) E_1 and E_2 are the magnitude of electric field due to upper hemispherical shell and lower spherical shell respectively, whose components along the diameter and its perpendicular direction are shown in the figure. By the combination of two hemispherical shells: At $P, E_{11} = E_{21}$ and $E_{10} = E_{20} = 0$

Net electric field must be zero.



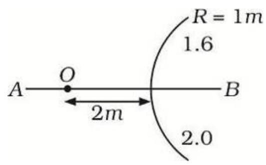
17.(B)

18.(D) In steady state current through the branch having only 4Ω resistor is $i = \frac{15}{8}A$

Now charge on any capacitor is $q_{\mu c}$

$$\frac{q}{3/2} = \frac{15}{8}(4)$$

$$q = \frac{60}{8} \times \frac{3}{2} = \frac{90}{8} = \frac{45}{4}\mu C$$

19.(A) 

$I_1 \Rightarrow \frac{1.6}{v} - \frac{1}{-2} = \frac{1.6-1}{+1} \Rightarrow v_1 = 16$; $I_2 \Rightarrow \frac{2.0}{v} - \frac{1}{-2} = \frac{2-1}{1} \Rightarrow v_2 = 4$

$|v_1 - v_2| = 12\ m$

20.(A) $\frac{q_1}{a^2} = \frac{q_2}{b^2} = \frac{Q - q_1 - q_2}{C^2}$; $q_1 = \frac{a^2}{b^2} q_2$; $q_2 = \frac{b^2}{C^2} Q - \frac{b^2}{C^2} q_1 - \frac{b^2}{C^2} q_2$

$\Rightarrow q_2\left(1 + \frac{b^2}{C^2}\right) = \frac{b^2}{C^2} Q - \frac{b^2}{C^2} q_1$ $\quad \Rightarrow \frac{b^2}{a^2} q_1 \left(1 + \frac{b^2}{C^2}\right) = \frac{b^2}{C^2} Q - \frac{b^2}{C^2} q_1$

$\Rightarrow \left(\frac{b^2}{a^2} + \frac{b^4}{a^2 c^2}\right) q_1 + \frac{b^2}{C^2} q_1 = \frac{b^2}{C^2} Q$ $\quad \Rightarrow \frac{b^2}{a^2} q_1 \left(1 + \frac{b^2}{C^2} + \frac{a^2}{C^2}\right) = \frac{b^2}{C^2} Q$

$q_1 = \frac{\frac{a^2}{C^2} Q}{\frac{C^2 + b^2 + a^2}{C^2}}$; $q_1 = \frac{a^2}{a^2 + b^2 + c^2} Q$; $q_2 = \frac{b^2}{a^2 + b^2 + c^2} Q$

$Q - q_1 - q_2 = \frac{c^2}{a^2 + b^2 + c^2} Q$; $\quad V = \frac{kq_1}{a} + \frac{kq_2}{b} + \frac{k(Q - q_1 - q_2)}{C}$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{a}{a^2 + b^2 + c^2} + \frac{b}{a^2 + b^2 + c^2} + \frac{c}{a^2 + b^2 + c^2}\right] = \frac{Q}{4\pi\varepsilon_0}\left(\frac{a + b + c}{a^2 + b^2 + c^2}\right)$$

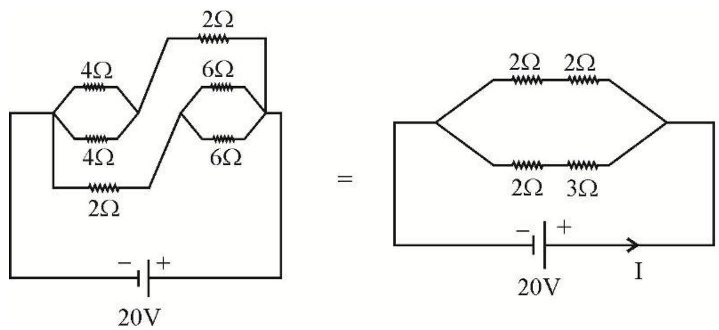
NUMERICAL TYPE

1.(80) $D = 1m, d = 5 \times 10^{-3} m$

Shift $= \frac{D}{d}\left[t_1(\mu_1 - 1) - t_2(\mu_2 - 1)\right]$

$$= \frac{1}{5 \times 10^{-3}}\left[2(1.5 - 1) - 1.5(1.4 - 1)\right] \times 10^{-3} = \frac{0.4}{5} = 0.08m$$

2.(9) The equivalent circuit is:



$R = \frac{20}{9}\Omega \quad \therefore \quad I = \frac{V}{R} = \frac{20}{\frac{20}{9}} A$

$I = 9A$

3.(2) Let E = Energy falling on the surface per second = 30 J

Momentum of photons = $p = \frac{h}{\lambda} = \frac{h}{(d_1 v)} = \frac{hv}{c} = \frac{E}{c}$

On reflection,

Change in momentum per second $= 2p = \frac{2E}{c}$

We know that,

Change in momentum per second = force $F = \frac{2E}{c} = \frac{2 \times 30}{3 \times 10^8} = 2 \times 10^{-7} N$

4.(5) Force between the plates, $F = \frac{Q^2}{2\varepsilon_0 A} = \frac{Q^2}{2Cd} = \frac{U}{d} = \frac{0.01}{(0.002)} = 5N$

5.(15) $0.5 = 0.4i \times 0.5 \Rightarrow i = 2.5A$

$\Rightarrow 8 - 2 \times 2.5 - v(0.4)(0.5) = 0 \Rightarrow v = 15 m/s$

6.(50) $\frac{X}{Y} = \frac{20}{80}$; $\frac{4X}{Y} = \frac{x}{100 - x} \quad \Rightarrow \quad x = 50\ cm$

7.(16) For first refraction $u = -x,\ v = -20,\ R = -40$

$$\frac{1.5}{-20} - \frac{1}{-x} = \frac{1.5 - 1}{-40} \quad \Rightarrow \quad x = 16cm$$

Light ray shall fall perpendicular on silvered surface.

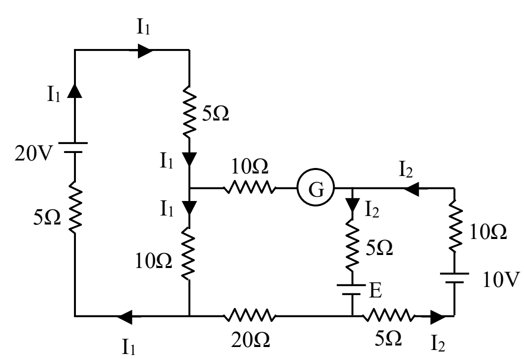
8.(12) $\frac{\Delta x}{x} = \frac{3\Delta a}{a} + \frac{2\Delta b}{b} + \frac{\Delta c}{c} \quad \Rightarrow \quad \frac{\Delta x}{x} \times 100 = (3 \times 2) + (2 \times 1) + (4) = 12\%$

9.(10) For no deflection in galvanometer there should not be any current in 20Ω resistance also, and current will be flowing as show.

$$I_1 \times 10 = I_2 \times 5 + \varepsilon$$

$$\Rightarrow \quad \frac{20}{20} \times 10 = \frac{10 - \varepsilon}{20} \times 5 + \varepsilon$$

$$\Rightarrow \quad \varepsilon = 10\ volts$$



10.(5) Assume a solid sphere without cavity.
Potential at A due to this solid sphere

$$\Rightarrow \qquad V_A' = \frac{3}{2}\cdot\frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{4}{3}\pi R^3\rho\right)}{R} = \frac{\rho R^2}{2\varepsilon_0}$$

Now consider the cavity filled with negative charge $\qquad V_A'' = \frac{1}{4\pi\varepsilon_0}\frac{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3(-\rho)}{\frac{R}{2}} = \frac{-\rho R^2}{12\varepsilon_0}$

Now net value for the solid sphere with the cavity can be given by superposition of the above two cases.

Hence, $V_A = V_A' + V_A'' = \frac{\rho R^2}{\varepsilon_0}\left(\frac{1}{2}-\frac{1}{12}\right) = \frac{5\rho R^2}{12\varepsilon_0}$

CHEMISTRY

SECTION-1

1.(A) In DNA, Adenine forms hydrogen bonds with thymine and cytosine forms hydrogen bonds with guanine.

$A \rightarrow T$

$G \rightarrow C$

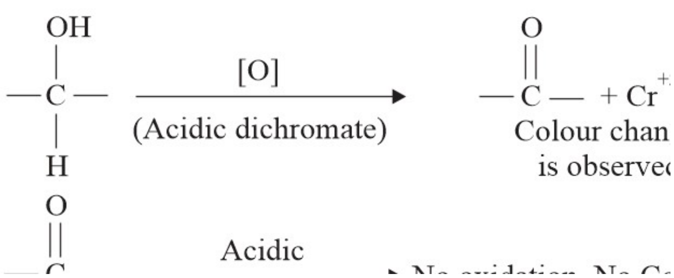
2.(B) Double strand helix structure – H – bond

Tripeptide – Amide bond

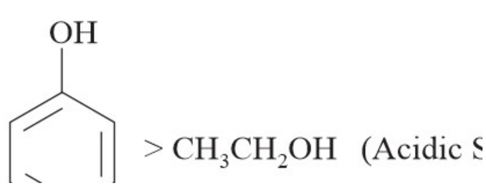
Disaccharide – Glycosidic linkage

Dinucleotide – Phosphodiester linkage

3.(D)



4.(D)



Hydroxyl group attached to an aromatic ring is highly acidic due to resonance than one in which hydroxyl group is attached to an alkyl group.

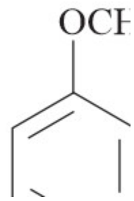
pKa : Ethanol : $\rightarrow$ 15.9

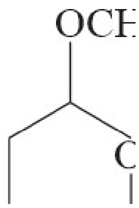
Phenol : $\rightarrow$ 10

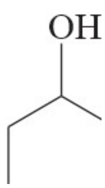
O-cresol $\rightarrow$ 10.2

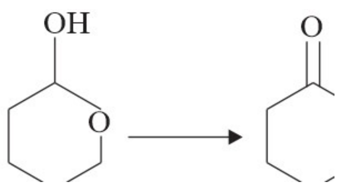
P-cresol $\rightarrow$ 10.2

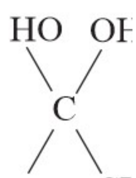
5.(C)

 $\rightarrow$ undergoes Friedel-Crafts reaction

 $\rightarrow$ reactive under acidic condition

 $\rightarrow$ alcohols give red colouration with ceric ammonium nitrate.

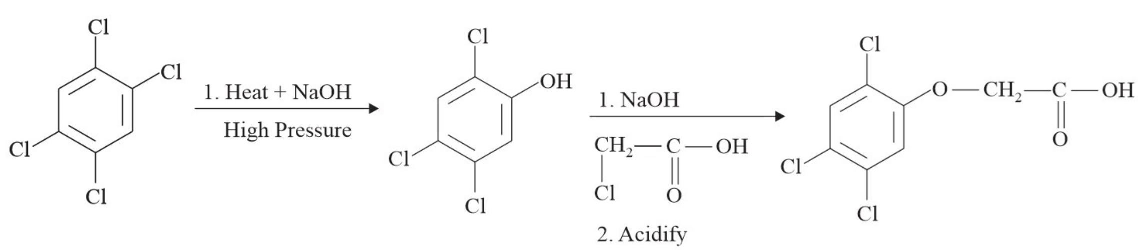
 $\rightarrow$ positive Tollen's test

6.(D) $CH_3-\underset{\underset{O}{\|}}{C}-CH_3 \xrightarrow{OH^{\ominus}/H_2O}$ 

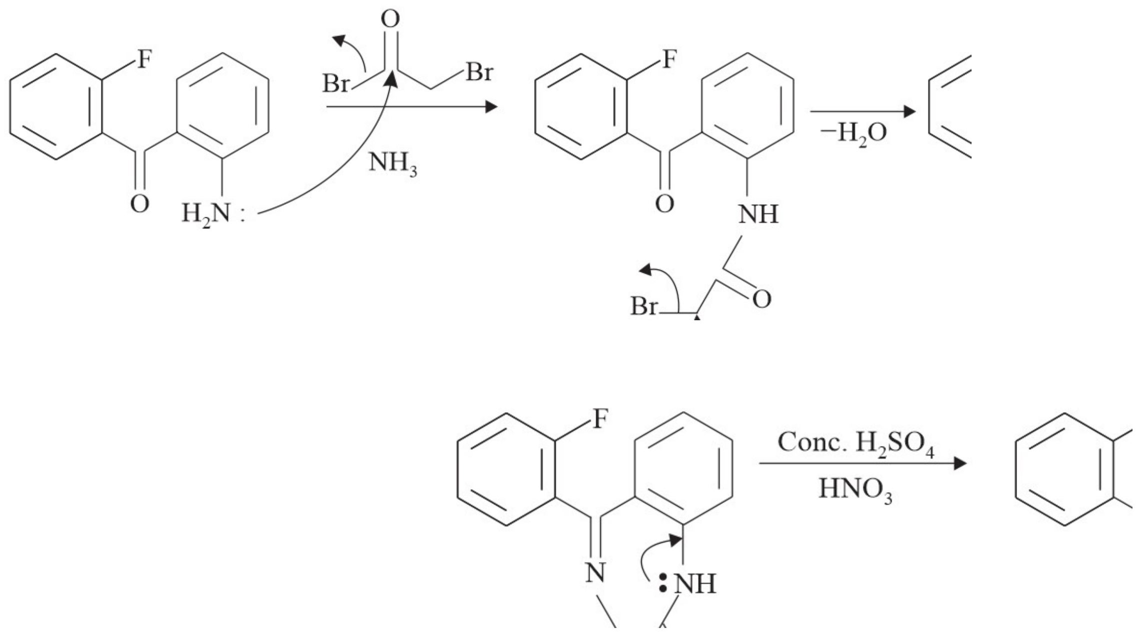
Aldol reaction : $CH_3-\underset{\underset{O}{\|}}{C}-CH_2-\overset{\overset{OH}{|}}{\underset{\underset{CH_3}{|}}{C}}-CH_3$

Aldol condensation : $CH_3-\underset{\underset{O}{\|}}{C}-CH=\underset{\underset{CH_3}{|}}{C}-CH_3$

7.(B)



8.(A)



9.(C) Greater size of P compared to N results in decreased overlap in pi-bonds.

10.(C) Ar (B.P.) = 87.2 K

F_2 (B.P.) = 84.9 K

Ar and F_2 have comparable molar mass.

11.(C) Oganesson is group 18 elements ; Tennessine is group 17 element.

12.(C) Longest wavelength $\rightarrow$ Energy gap between t_{2g} and e_g level will be lowest.

Fluorine is the weak field ligand among all.

In case of weak field, splitting is minimum $[CoF_6]^{3-}$.

13.(A) $M(CO)_2BrCl$ (Two isomeric)

C.N. = 4 $\rightarrow$ square planer

Tetrahedral

The complex must be square planar as there will be no geometrical isomerism in the case of tetrahedral.

14.(B) Aq. KI

$K^+, H^+, H_2O \rightarrow$ cathode

$I^\ominus, OH^\ominus, H_2O \rightarrow$ anode

At anode :

$F^\ominus < NO_3^\ominus < H_2O < Cl^\ominus < Br^\ominus < I^\ominus$ (Order of oxidation)

Anode : $I^\ominus \rightarrow I_2$

15.(A) $Hg_2Cl_2 \rightleftharpoons Hg_2^{2+} + 2Cl^\ominus \quad K_{SP} = [Hg_2^{2+}][Cl^-]^2$

$Hg_2Cl_2(s) + 2e^- \longrightarrow 2Hg(\ell) + 2Cl^\ominus(aq) \quad E° = 0.31\ V$

$2Hg(\ell) \longrightarrow Hg_2^{2+} + 2e^- \quad E° = -0.80\ V$

$Hg_2Cl_2 \longrightarrow 2Cl^\ominus + Hg_2^{2+} \quad E° = -0.49\ V$

$$\Delta G^\circ = -nFE^\circ = -RT \text{ In } k_{SP}$$

$$\text{In } k_{sp} = \frac{nFE^\circ}{RT}$$

$$\text{In } k_{sp} = \frac{2 \times 96500 \times (-0.49)}{8.314 \times 298}$$

$$\text{In } k_{sp} = -38.17$$

$$\log\ k_{sp} = -16.57$$

$$k_{sp} = 2.6 \times 10^{-17}$$

16.(D) Graduated measuring cylinder.

17.(C) $Ag^{+} + aq.HCl \rightarrow AgCl(\downarrow)$ (white ppt)

$Pb^{2+} + aq.HCl \rightarrow PbCl_2(\downarrow)$ (white ppt)

$PbCl_2$ can dissolve in hot water

18.(D) Homogenous catalyst

19.(D) Applying SSA:

$$\frac{d(\text{Intermediate})}{dt} = O = \frac{d[(CH_3)_3C^{\oplus}]}{dt}$$

$$r = k_2[(CH_3)_3C^{+}][N_3^{-}]$$

$$\frac{d[(CH_3)_3C^{+}]}{dt} = k_1[tBuBr] - k_{-1}[(CH_3)_3C^{+}][Br^{-}] - k_2[(CH_3)_3C^{\oplus}][N_3^{-}]$$

$$\frac{d[(CH_3)_3C^{+}]}{dt} = O$$

$$[(CH_3)_3C^{+}] = \frac{k_1[tBuBr]}{k_{-1}[Br^{-}] + k_2[N_3^{-}]}$$

$$r = \frac{k_2 \cdot k_1[tBuBr][N_3^{-}]}{k_{-1}[Br^{-}] + k_2[N_3^{-}]}$$

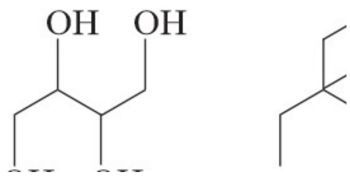
20.(A) Lanthanoid contraction:

Presence of the 14 Lanthanoids between Pd & Pt makes Pt unexpectedly small because Z_{eff} will increase.

SECTION-2

1.(9) See the structure of sucrose. [NCERT]

2.(4) Degree of unsaturation = O

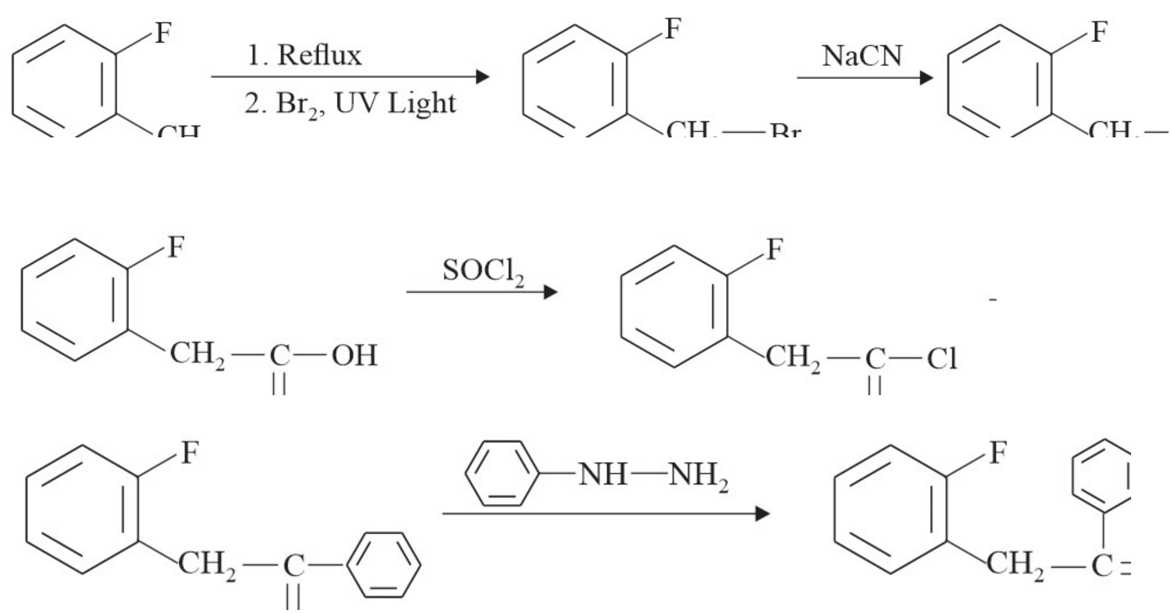


2 chiral centre with symmetrical end

$= 2^{n-1} + 2^{n/2-1} = 2^{2-1} + 2^{2/2-1} = 2 + 1 = 3$

Total = 3 + 1 = 4

3.(3)



x = 1 ; y = 2 ; x + y = 3

4.(2) 0.02279 L titrant × 0.3540 mol L^{-1} = 8.068×10^{-3} mol chloride

100.0 mL gas has n = (1.000 bar)(0.1000 L) / (298.15 K) (0.08314 L bar mol^{-1} K^{-1})

$= 4.034 \times 10^{-3}$ mol A

(8.068×10^{-3} mol chloride) / (4.034×10^{-3} mol A) = 2.000 mol Cl/mol A

5.(6433) $k_2[PtCl_4]$

$Pt^{2+} + 2e^- \longrightarrow Pt$

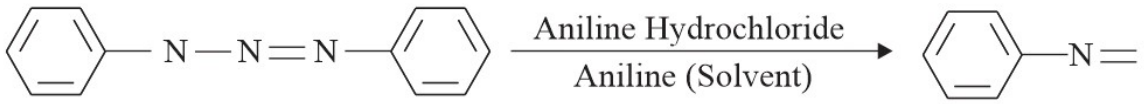
$2F \longrightarrow 200\ g$

$200g \longrightarrow 2 \times 96500\ C$

$1g \longrightarrow \frac{2 \times 96500}{200} C$

$\frac{2 \times 96500}{200} = 0.15 \times t \ ; \ t = \frac{2 \times 96500}{0.15 \times 200} = \frac{193000}{30} = 6433.33$

6.(6)



X Y

Single bond : $N-N=1$, $C-N=2$

Single bond : $N-N=0$, $C-N=2$

Sum = 3 + 3 = 6

7.(3) $\underset{0.001}{Cu^{2+}} + \underset{x}{4NH_3} \rightleftharpoons \underset{0.999}{[Cu(NH_3)_4]^{2+}}$ $k_f = 1.1\times10^{13}$

$$k_f = \frac{[Cu(NH_3)_4]^{2+}}{[Cu^{2+}][NH_3]^4}$$

$$1.1\times10^{13} = \frac{0.999}{0.001\times(x)^4}$$

$$x^4 = 90.81\times10^{-12}$$

$$x = 3.08\times10^{-3}$$

8.(93) $-\frac{1}{5}\frac{d[Br]}{dt} = \frac{1}{3}\frac{d[Br_2]}{dt}$

$$-\frac{d[Br^-]}{dt} = \frac{5}{3}\times0.056\ Ms^{-1}$$

$$= \frac{0.28}{3}$$

$$= 93.3\times10^{-3}\ Ms^{-1}$$

9.(4) **A, B, D, E**

Energy difference between 2p-oxide and (4d-5d) orbitals of transition metals is very large which lies in UV-region.

$F^{\ominus}$ is a weak field Ligand.

$CN^{\ominus}$ is a strong field Ligand.

10.(1) Molar Mass $= \frac{7.05\times2.000\times1000}{0.5\times31.6}$

$$= \frac{14100}{15.8}$$

$$= 892.40$$

$$= 8.924\times10^2$$

$$= 9\times10^2$$

In case of multiplication/division, the number of significant figures in the result will be equal to the significant figure of that multiplied number having least significant figure.

MATHEMATICS

SECTION-1

1.(B) We have $A^2 = (AB)A = A(BA) = AB = A$

Similarly $B^2 = B$: $(A + B)^2 = A^2 + B^2 + AB + BA = A + B + A + B = 2(A + B)$

$\Rightarrow \quad |A + B|^2 = 8|A + B| \Rightarrow |A + B| = 8$ and $(A - B)^2 = A^2 + B^2 - AB - BA = A + B - A - B = O$

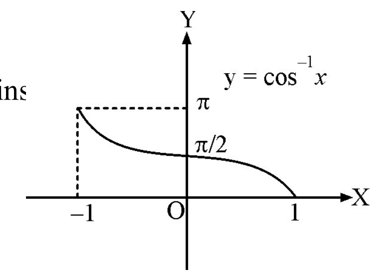
$\Rightarrow \quad |A - B| = 0$

2.(C) We have, z = 0 for the point where the line intersects the curve

$\therefore \quad \frac{x-2}{3} = \frac{y+1}{2} = \frac{0-1}{-1} \quad \Rightarrow \quad x = 5$ and $y = 1$

Putting these values in $xy = c^2 \quad \Rightarrow \quad 5 = c^2 \Rightarrow c = \pm\sqrt{5}$

3.(A) $f(x) = \underset{(x \in [-1,1])}{\cos^{-1} x} + \underset{(x \in R)}{\tan^{-1} x}$

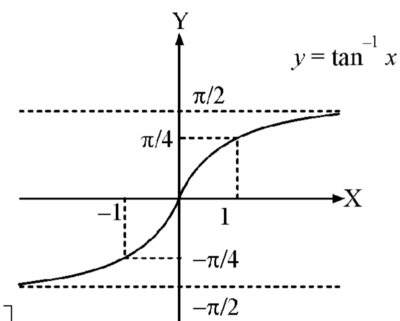


$f(x)$ is defined only for $x \in [-1, 1]$ (common to both domains

$\forall\ x \in [-1, 0]$, $\cos^{-1} x$ decreases form π to $\frac{\pi}{2}$

and $\tan^{-1} x$ increases from $\frac{-\pi}{4}$ to 0.

Hence, $\forall\ x \in [-1, 0]$, $\cos^{-1} x + \tan^{-1} x \in \left[\frac{\pi}{2}, \frac{3\pi}{4}\right]$



$\forall\ x \in [0, 1]$, $\cos^{-1} x$ decreases from $\frac{\pi}{2} to\ 0$

and $\tan^{-1} x$ increases from 0 to $\frac{\pi}{4}$

Hence, $\forall\ x \in [0,1]$, $\cos^{-1} x + \tan^{-1} x \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right]$

Combining both cases, we get : $\forall\ x \in [-1, 1]$, $f(x) \in \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

4.(D) $I_{11} = \int_0^1 (1 - x^5)^{11}\, dx$

$= \int_0^1 (1 - x^5)(1 - x^5)^{10}\, dx \ = I_{10} - \int_0^1 x^5 (1 - x^5)^{10}\, dx$

$= I_{10} + \int_0^1 \frac{x}{5}(1 - x^5)^{10}(-5x^4)\, dx$

Integrating by parts,

$$I_{11} = I_{10} + \left(\frac{x(1 - x^5)^{11}}{5.11}\right)_0^1 - \frac{1}{55}\int_0^1 (1 - x^5)^{11}\, dx$$

$$= I_{10} - \frac{I_{11}}{55} = \left(1 + \frac{1}{55}\right) I_{11} = I_{10} \qquad \Rightarrow \qquad \frac{I_{10}}{I_{11}} = \frac{56}{55}$$

5.(B) P(3 or more correct) = (P3 correct, 2 wrong) + P(4 correct, 1 wrong) + P(5 correct, 0 wrong)

$$= \frac{^5C_3}{\lfloor 5} + \frac{0}{\lfloor 5} + \frac{1}{\lfloor 5} = \frac{11}{120}$$

6.(B) $\lim\limits_{x \to 0} \left(\frac{f^2(a+x)}{f(a)} \right)^{1/x}$

$= e^{\lim\limits_{x \to 0} \left(\frac{f^2(a+x) - f(a)}{xf(a)} \right)}$

$= e^{\lim\limits_{x \to 0} \frac{f^2(a+x) - (f(a))^2}{x}}$, $= e^4$

7.(C) Given $f(2x) - f(x) = x.$

Replacing $x \, by \, x/2,$ we have $f(x) - f\left(\frac{x}{2}\right) = \frac{x}{2}.$

Replacing x by $\frac{x}{2}$ repeatedly and adding, we get

$\left(f(x) - f\left(\frac{x}{2}\right)\right) + \left(f\left(\frac{x}{2}\right) - f\left(\frac{x}{4}\right)\right) + \ldots + \left(f\left(\frac{x}{2^{n-1}}\right) - f\left(\frac{x}{2^n}\right)\right)$

$= \frac{x}{2} + \frac{x}{2^2} + \frac{x}{2^3} + \ldots + \frac{x}{2^n}$

$\Rightarrow \quad f(x) - f\left(\frac{x}{2^n}\right) = \frac{x}{2} + \frac{x}{2^2} + \frac{x}{2^3} + \ldots. + \frac{x}{2^n}$

$\therefore \quad \lim\limits_{n \to \infty} \left(f(x) - f\left(\frac{x}{2^n}\right)\right) = \frac{\frac{x}{2}}{1 - \frac{1}{2}} \quad \Rightarrow \quad f(x) - f(0) = x$

$\left(\because \; f(x) \text{ is continuous } \lim\limits_{n \to \infty} f\left(\frac{x}{2^n}\right) = f(0) = 1 \right)$

$\Rightarrow \quad f(x) = x + f(0) = x + 1 \quad \Rightarrow \quad f(3) = 4$

8.(A) $I = \int (\sin(100x + x).(\sin x)^{99} \, dx$

$= \int ((\sin(100x) \cos x + \cos 100x. \sin x)(\sin x)^{99}) dx$

$\int \underbrace{\sin(100x)}_{I} \underbrace{\cos x.(\sin x)^{99}}_{II} dx + \int \cos(100x).(\sin x)^{100} dx$

$= \frac{\sin(100x)(\sin x)^{100}}{100} - \frac{100}{100} \int \cos(100x)(\sin x)^{100} dx + \int \cos(100x)(\sin x)^{100} dx$

$= \frac{\sin(100x)(\sin x)^{100}}{100} + C$

9.(D) $|\vec{a}|=1, |\vec{b}|=4, \vec{a}.\vec{b}=2$

$\vec{c}=\left(2\vec{a}\times\vec{b}\right)-3\vec{b}$

$\Rightarrow \quad \vec{c}+3\vec{b}=2\vec{a}\times\vec{b} \qquad \because \quad \vec{a}.\vec{b}=2$

$\Rightarrow \quad |\vec{a}|.|\vec{b}|\cos\theta=2 \qquad \Rightarrow \quad \cos\theta=\dfrac{2}{|\vec{a}.|\vec{b}|}=\dfrac{2}{4}$

$\Rightarrow \quad \cos\theta=\dfrac{1}{2} \qquad \therefore \quad \theta=\dfrac{\pi}{3}$

$\Rightarrow \quad |\vec{c}+3\vec{b}|^2=|2\vec{a}\times\vec{b}|^2 \qquad \Rightarrow \quad |\vec{c}|^2+9|\vec{b}|^2+2\vec{c}.3\vec{b}=4|\vec{a}|^2|\vec{b}|^2\sin^2\theta$

$\Rightarrow \quad |\vec{c}|^2+144+6\vec{b}.\vec{c}=48$

$\Rightarrow \quad |\vec{c}|^2+96+6(\vec{b}.\vec{c})=0 \qquad \ldots \text{(i)}$

$\because \quad \vec{c}=2\vec{a}\times\vec{b}-3\vec{b} \qquad \Rightarrow \quad \vec{b}.\vec{c}=0-3\times16 \qquad \therefore \quad \vec{b}.\vec{c}=-48$

Putting value of $\vec{b}.\vec{c}$ in (i), we get

$|\vec{c}|^2+96-6\times48=0 \qquad \Rightarrow \quad |\vec{c}|^2=48\times4 \qquad \Rightarrow \quad |\vec{c}|^2=192$

Again, putting the value of $|\vec{c}|$ in (i), we get

$192+96+6|\vec{b}|.|\vec{c}|\cos\alpha=0$

$\Rightarrow \quad 6\times4\times8\sqrt{3}\cos\alpha=-288 \qquad \Rightarrow \quad \cos\alpha=-\dfrac{288}{6\times4\times8\sqrt{3}}=\dfrac{3}{2\sqrt{3}}$

$\Rightarrow \quad \cos\alpha=-\dfrac{\sqrt{3}}{2} \qquad \therefore \quad \alpha=\dfrac{5\pi}{6}$

10.(C) According to the given conditions.

$c_1^2+c_2^2+c_3^2=1, a\,.\,c=0, b\,.\,c=0$ and $\dfrac{\sqrt{3}}{2}=\cos\dfrac{\pi}{6}=\dfrac{a_1b_1+a_2b_2+a_3b_3}{\sqrt{a_1^2+a_2^2+a_3^2}\sqrt{b_1^2+b_2^2+b_3^2}}$

Thus $\quad a_1c_1+a_2c_2+a_3c_3=0,\ b_1c_1+b_2c_2+b_3c_3=0$ and $\dfrac{\sqrt{3}}{2}\left(a_1^2+a_2^2+a_3^2\right)^{1/2}\left(b_1^2+b_2^2+b_3^2\right)^{1/2}$

$=a_1b_1+a_2b_2+a_3b_3$

Now, $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}^2=\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

$$=\begin{vmatrix} a_1^2+a_2^2+a_3^2 & a_1b_1+a_2b_2+a_3b_3 & a_1c_1+a_2c_2+a_3c_3 \\ a_1b_1+a_2b_2+a_3b_3 & b_1^2+b_2^2+b_3^2 & b_1c_1+b_2+c_2+b_3c_3 \\ a_1c_1+a_2c_2+a_3c_3 & b_1c_1+b_2c_2+b_3c_3 & c_1^2+c_2^2+c_3^2 \end{vmatrix}$$

$$=\begin{vmatrix} a_1^2+a_2^2+a_3^2 & a_1b_1+a_2b_2+a_3b_3 & 0 \\ a_1b_1+a_2b_2+a_3b_3 & b_1^2+b_2^2+b_3^2 & 0 \\ 0 & 0 & 1 \end{vmatrix}=\left(a_1^2+a_2^2+a_3^2\right)\left(b_1^2+b_2^2+b_3^2\right)-\left(a_1b_1+a_2b_2+a_3b_3\right)^2$$

$$=\left(a_1^2+a_2^2+a_3^2\right)\left(b_1^2+b_2^2+b_3^2\right)-\frac{3}{4}\left(a_1^2+a_2^2+a_3^2\right)\left(b_1^2+b_2^2+b_3^2\right)$$

$$=\frac{1}{4}\left(a_1^2+a_2^2+a_3^2\right)\left(b_1^2+b_2^2+b_3^2\right)$$

11.(D) $\dfrac{xdx+ydy}{ydx-xdy}=x^2+2y^2+\dfrac{y^4}{x^2}=\dfrac{\left(x^2+y^2\right)^2}{x^2}$

$\Rightarrow \quad \dfrac{xdx+ydy}{\left(x^2+y^2\right)^2}+\dfrac{xdy-ydx}{x^2}=0 \quad \Rightarrow \dfrac{d\left(x^2+y^2\right)}{\left(x^2+y^2\right)^2}+2d\left(\dfrac{y}{x}\right)=0 \quad \Rightarrow \quad 2\dfrac{y}{x}-\dfrac{1}{\left(x^2+y^2\right)}=C$

12.(A) Put $1+x^{4/5}=t^2$

$\Rightarrow \dfrac{4}{5}\cdot\dfrac{1}{x^{1/5}}dx=2tdt \qquad \Rightarrow I=\int\dfrac{2tdt}{(4/5)t} \qquad \Rightarrow I=\dfrac{5}{2}t+c=\dfrac{5}{2}\left(1+x^{4/5}\right)^{1/2}+C$

13.(C) $y=x+\tan x$

$x=y+\tan y \ \left(y=g(x)\right)$

$1=y'+(\sec^2 y)y'$

$y'=\dfrac{1}{1+\sec^2 y}=\dfrac{1}{2+\tan^2 y}=\dfrac{1}{2+(y-x)^2}$

14.(A) $\dfrac{dy}{dx}=\dfrac{y(x-y\ln y)}{x(x\ln x-y)}$

$\Rightarrow \quad x^2\ln xdy-xydy=xydx-y^2\ln ydx$

On dividing by x^2y^2, we get, $\dfrac{\ln x}{y^2}dx-\dfrac{1}{xy}dy=\dfrac{1}{xy}dx-\dfrac{\ln y}{x^2}dx$

$\Rightarrow \quad \left(\ln x\left(-\dfrac{1}{y^2}dy\right)+\dfrac{1}{xy}dx\right)+\left(\ln y\left(-\dfrac{1}{x^2}dx\right)+\dfrac{1}{xy}dy\right)=0 \qquad \Rightarrow \qquad d\left(\dfrac{\ln x}{y}\right)+d\left(\dfrac{\ln y}{x}\right)=0$

On integrating, we get $\dfrac{\ln x}{y}+\dfrac{\ln y}{x}=c \quad \Rightarrow \quad \dfrac{x\ln x+y\ln y}{xy}=c$

15.(A) For the domain of the function $f(x)=\sqrt{\sin^{-1}x+\cos^{-1}\sqrt{1-x^2}}$

$\sin^{-1}x+\cos^{-1}\sqrt{1-x^2}\ge 0 \ \Rightarrow \ x\in[-1,1]$

And also $1-x^2\ge 0 \Rightarrow x\in[-1,1] \quad \Rightarrow \quad x\in[-1,1]$ is the domain.

16.(C) Put $\ln x=t$

$$I=\int e^t\left(\frac{t-1}{t^2+1}\right)^2 dt=\int e^t\left(\frac{1}{t^2+1}-\frac{2t}{\left(t^2+1\right)^2}\right)dt=\frac{e^t}{t^2+1}+c=\frac{x}{(\ln x)^2+1}+c.$$

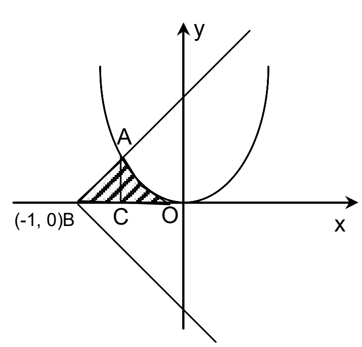
17.(A) $|y|=x+1$, $x^2=8y$, x-axis

Required area OABCO = area of ΔABC + area of AOCA

Coordinate of $A(4-2\sqrt{6},\ 5-2\sqrt{6})$

Required area $=\dfrac{1}{2}(5-2\sqrt{6})^2+\displaystyle\int_{4-2\sqrt{6}}^{0}\dfrac{x^2}{8}dx$

$=\dfrac{\left(5-2\sqrt{6}\right)^2}{2}+\dfrac{\left(4-2\sqrt{6}\right)^3}{24}$



18.(A) The line is $\frac{x-1}{2}=\frac{y}{-1}=\frac{z+2}{1}=r$. There is no point of intersection of line and the plane hence line is parallel to the plane so direction cosines of the projected line remain same.

19.(C) $0.7 = P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B) - P(A).\ P(B) = 0.2 + P(B) - 0.2P(B)$

$\Rightarrow \quad P(B) = \frac{0.5}{0.8} = \frac{5}{8} \quad \Rightarrow \quad P(B') = 3/8.$

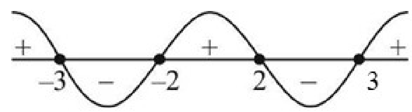
20.(C) Given that 5 and 6 have appeared on two of the dice the sample space reduces to
$6^4 - 2 \times 5^4 + 4^4$ (inclusion exclusion principle)

Also favourable cases are 4! = 24. So the required probability is $\frac{24}{302}=\frac{12}{151}$.

SECTION-2

1.(18) $x^4+36 \le 13x^2 \Rightarrow x^4-13x^2+36 \le 0$

$\Rightarrow \quad (x-3)(x+3)(x-2)(x+2) \le 0$



Using wavy curve method, $x \in [-3, -2] \cup [2, 3]$

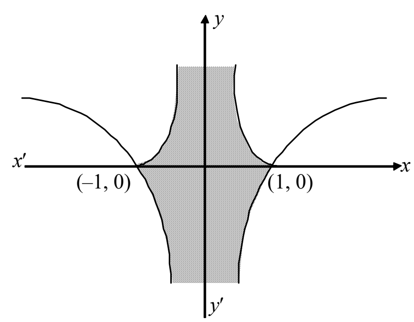
Now, $f(x) = x^3 - 3x$

$\Rightarrow \quad f'(x) = 3x^2 - 3 > 0$ for all $x \in [-3, -2] \cup [2, 3]$

$\therefore \quad f(x)$ increases as $x \in [-3, -2] \cup [2, 3]$

$\therefore \quad$ Maximum value of $f(x) = \max\{f(-2), f(3)\} = 18$

2.(4) Required area $= 4\int_0^1 |ln\, x|\, dx = -4\int_0^1 ln\, x\, dx$



$(\because x \in (0, 1), \ln x < 0)$

$= -4\left[x \ln x - x\right]_0^1 = -4(-1) = 4$

$(\because x \ln x = 0, x \to 0)$

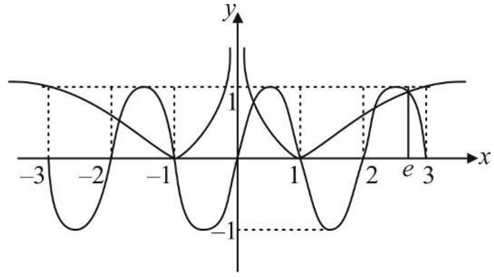
3.(5050) $\quad f(x) = \prod_{i=1}^{100} (x-i)^{i(101-i)}$

Taking log both sides, we have log $(f(x)) = \sum_{i=1}^{100} i(101-i)\log(x-i)$

Differentiating w.r.t. x, we get

$$\frac{1}{f(x)}.f'(x) = \sum_{i=1}^{100} \frac{i(101-i)}{(x-i)} \quad \Rightarrow \quad \frac{f'(101)}{f(101)} = \sum_{i=1}^{100} i\left(\frac{101-i}{101-i}\right) = 5050$$

4.(6) The graph of $y = \sin \pi x$ and $y = |\ln|x||$ intersect at 6 points (four positive and two negative roots) as clear from the given figure.



5.(5) $f(x)f(y)+2=f(x)+f(y)+f(xy)\forall x, y\in R$

Put $y=\frac{1}{x}$; $f(x)f\left(\frac{1}{x}\right)+2=f(x)+f\left(\frac{1}{x}\right)+f(1)$; $f(x)f\left(\frac{1}{x}\right)=f(x)+f\left(\frac{1}{x}\right)$

Let $f(x)=1+x^n$

$\because$ $f'(1)=2$ $\therefore$ $n=2$ $\therefore$ $f(x)=1+x^2$

6.(9) $f(x)=x^2|\sin\pi x|$

Is non differentiable at all integers except $x=0$

At $x=n, n\in Z$

RHD $=\lim\limits_{h\to 0}\frac{(n+h)^2|\sin\pi(n+h)|-0}{h}=\lim\limits_{h\to 0}(n+h)^2\frac{\sin\pi h}{h}$, $=n^2\pi$

LHD $=\lim\limits_{h\to 0}\frac{(n-h)^2|\sin(\pi(n-h))|}{-h}=\lim\limits_{h\to 0}(n-h)^2\frac{\sin\pi h}{-h}$, $=-n^2\pi$

RHD $\neq$ LHD

But for $x=0$

RHD = LHD = 0

7.(63) $\because$ A is idempotent

$\Rightarrow$ $A^n=A$; $n\in N$

$\therefore$ $(I+A)^6=(2^6-1)A+I=63A+I$ $\therefore$ $k=63$

8.(8) $|A^{-1}\cdot Adj(B^{-1})Adj(2A^{-1})|=|A^{-1}||Adj(B^{-1})||Adj(2A^{-1})|$

$=\frac{1}{|A|}|B^{-1}|^2|2A^{-1}|^2$ (using $|Adj\ A|=|A|^{n-1}, |A^{-1}|=\frac{1}{|A|}, |kA|=k^n|A|$)

$=\frac{1}{|A|}\frac{1}{|B|^2}\cdot 2^6\frac{1}{|A|^2}=\frac{2^6}{2^3\times 1}=2^3=8$

9.(7) Let A be a point on the given line such that

Line PA is || to the plane

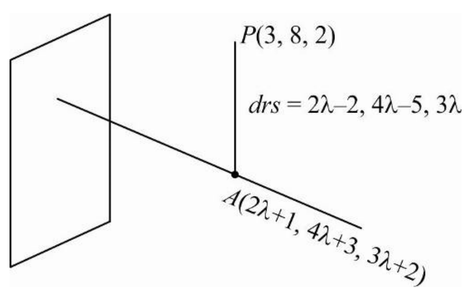
$\therefore$ Normal of plane in $\perp$ to PA

$3(2\lambda-2)+2(4\lambda-5)-2(3\lambda)=0$

$\Rightarrow$ $\lambda=2$

$\therefore$ $A\equiv(5, 11, 8)$

$\therefore$ $AP=\sqrt{(5-3)^2+(8-11)^2+(2-8)^2}=7$



10.(1) $-\frac{\pi}{2}\le 2\tan^{-1}2x\le\frac{\pi}{2}$ $\Rightarrow$ $\frac{\pi}{4}\le\tan^{-1}2x\le\frac{\pi}{4}$

$\Rightarrow$ $-1\le 2x\le 1$ $\Rightarrow$ $-\frac{1}{2}\le x\le\frac{1}{2}$.